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Closed Bosonic String Field Theory at Quintic Order II: Marginal Deformations and Effective Potential

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Abstract

We verify that the dilaton together with one exactly marginal field, form a moduli space of marginal deformations of closed bosonic string field theory to polynomial order five. We use the results of this successful check in order to find the best functional form of a fit of quintic amplitudes. We then use this fit in order to accurately compute the tachyon and dilaton effective potential in the limit of infinite level. We observe that to order four, the effective potential gives unexpectedly accurate results for the vacuum. We are thus led to conjecture that the effective potential, to a given order, is a good approximation to the whole potential including *all* interactions from the vertices up to this order from the untruncated string field. We then go on and compute the effective potential to order five. We analyze its vacuum structure and find that it has several saddle points, including the Yang-Zwiebach vacuum, but also a local minimum. We discuss the possible physical meanings of these vacua.

Contents

1 Introduction

The main goal of this paper is to continue the search for a nonperturbative closed bosonic string vacuum. Although this search in the context of closed bosonic string field theory (CSFT) [1] originally started in [2, 3, 4], an important breakthrough came in a paper by Yang and Zwiebach [5] where it was realized that the ghost dilaton must be included in the string field in the universal basis. Using the solution of the quartic CSFT vertex [6], they found a nonperturbative vacuum, namely an extremum of the potential truncated to order four. Through an argument based on the low-energy effective action of the closed tachyon, dilaton and massless fields, they conjectured that a CSFT vacuum must have zero action. In another paper [7], they proposed that this vacuum corresponds to infinite string coupling and that the universe undergoes a big crunch when the tachyon has rolled to it.

The Yang-Zwiebach vacuum was subsequently studied with more accuracy in [8]. The CSFT action was still truncated to quartic order, but fields of level up to ten where included in the string field. The potential value at the vacuum was seen to converge to approximately -0.050 (in units where $\alpha' = 2$). It was then concluded that the quintic terms of the potential should be included in order to test the vanishing potential conjecture.

The quintic term of the CSFT action was calculated in [9]. The solution is numerical, it gives the Strebel differentials determining the local coordinates, everywhere in the reduced moduli space. This is a complicated calculation, which could fortunately be checked by verifying the flatness of the dilaton potential to order five; but we devote one section of this paper to a further check of this

solution. Namely we will calculate the effective potential of the dilaton and one exactly marginal field, to order five. This is the direct extension of a calculation done in [10, 11] to order four. As expected, we find that the effective potential is flat (within the uncertainty on the quintic terms), thereby successfully checking the quintic contact term solution.

As a level truncation analysis similar to the one done in [5, 8], would require, at order five, many contact terms that are still time-consuming to compute, we decided to focus instead on the effective tachyon and dilaton potential. We are able to integrate out massive fields up to level twelve; but in order to obtain the exact terms in the effective potential (those found after integrating out all levels) we must extrapolate the results to infinite level. We find that the fits used until now in the literature are unsatisfactory; we therefore spend a section looking for the best possible functional form of a fit, and we find a simple expression that gives good results when checking the flatness of the dilaton and marginal effective potential. We then go on and use this fit for the calculation of the effective tachyon and dilaton potential to order five. We first spend some time studying this potential to order four. This allows us to observe that the Yang-Zwiebach vacuum found from the effective potential, matches very accurately the solution found from the potential to quartic order with *all* interactions from a string field at a given level. This is surprising because the effective potential lacks most of the quartic contact terms which are included in the full quartic potential. We turn this observation into an approximate conjecture, essentially stating that this remains true at higher order. To order five, this would imply that the effective potential, which requires only the quintic contact terms $\kappa^2 V_{t^5}$, $\kappa^2 V_{t^4 d}$, $\kappa^2 V_{t^3 d^2}$, $\kappa^2 V_{t^2 d^3}$, $\kappa^2 V_{t d^4}$, and $\kappa^2 V_{d^5}$, is a good approximation to the potential to order five (which contains many many more contact terms).

From the analysis of the effective potential to order five, we find that the Yang-Zwiebach vacuum still exists to this order, and is shallower than to order four, giving evidence for the vanishing of the potential at the vacuum. An advantage of the effective potential is that it allows to check easily if a given extremum is a local minimum, maximum, or saddle point. We find that the Yang-Zwiebach vacuum is a saddle point. But at order five, we also find a local minimum. We discuss these implications in the last section.

At last, we want to look at the usual level truncation of the potential, as was done in [5, 8] to order four. We were able to compute only a few contact terms, namely those of total level not greater than four. Surprisingly, we see that once we introduce the term of level two, the Yang-Zwiebach vacuum is destroyed (and does not reappear at level four). Although this should be checked at higher level, we argue that the effective potential analysis should be more reliable than the standard level truncation.

This paper is structured as follows: In Section 2, we verify the flatness of the potential in the combined dilaton and marginal directions. We use the data computed there in order to find a good universal fit in Section 3. We can then proceed to the computation of the effective potential in Section 4. The level truncation analysis is done in Section 5, and the results are compared and discussed in Section 6, where some physical interpretations are also discussed. The technical details of the calculations of quintic contact terms are collected in Appendix A.

2 Combined dilaton and marginal deformations

There are two objectives in this section: We want to test further our computations of the quintic contact terms; and we want to verify that the effective potential of the dilaton together with an exactly marginal field, is flat. Our code that computes quintic contact terms [9] was already successfully checked by verifying the flatness of the dilaton effective potential at order five. This showed that the five-dilaton contact term $\kappa^2 V_{d^5}$ has been computed correctly; our code was thus seen reliable at least for the computation of contact terms of five identical states. Here we want to extend this check to the computations of terms involving two different kinds of states; this is in fact all that will be needed in the rest of this paper, either for the tachyon and dilaton effective potential which requires the contact terms of n tachyons and $5-n$ dilatons, or for the potential with quintic terms to level four, which requires the contact terms of four tachyons and one massive field. The computations of quintic terms of states not all equal, involve some (not difficult but not completely trivial) combinatorics and also some symmetry of the reduced moduli space. The technical details are explained in Appendix A. Concretely, we will verify the flatness of the effective potential of the ghost dilaton d and an exactly marginal field a. The dilaton is given by

$$d|D\rangle = d\,(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1})|0\rangle, \tag{2.1}$$

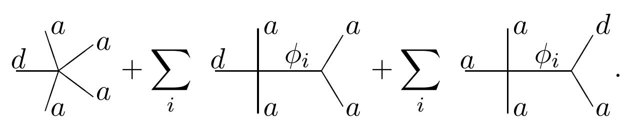
and the marginal field is

$$a|A\rangle = a\,\alpha^X_{-1}\bar{\alpha}^X_{-1} c_1 \bar{c}_1 |0\rangle, \tag{2.2}$$

where we have singled out one spacetime dimension X. Our analysis is the direct extension, to order five, of the analysis made by Yang and Zwiebach in [11]. There the authors showed that the contact terms $\kappa^2 V_{a^4}$ and $\kappa^2 V_{a^2 d^2}$ are canceled by the contributions from cubic interactions. In this section we will similarly show that the contact terms $\kappa^2 V_{a^4 d}$ and $\kappa^2 V_{a^2 d^3}$ are canceled by the contributions from cubic and quartic vertices.

We start with the effective term $\kappa^2 V^{\text{eff}}_{a^4 d}$ of four marginals and one dilaton. We write it diagrammatically as

$$-\,4!\,i\,\kappa^2 V^{\text{eff}}_{a^4 d} =$$



(2.3)

The easiest way to understand the coefficient in the left-hand side is to note that the right-hand side is an amplitude, and that to form an amplitude from a term in the potential one should include the combinatorial factor (here 4! is the number of ways to assign the four marginals) and a $-i$ (we are in Minkowski space, all vertices bring a factor $-i$ and the propagators bring a factor i). The internal fields ϕ_i are all the scalar fields, except for the marginal field and dilaton. More explicitly we construct the components of the closed string field $|\Phi\rangle = \sum_i \phi_i |\Phi_i\rangle$ in the Siegel gauge, from open fields $\tilde{\mathcal{O}}_j|0\rangle$ and $\tilde{\mathcal{O}}_k|0\rangle$ of same levels and arbitrary ghost numbers, provided they add up to two.

$$|\Phi_i\rangle = \left(\tilde{\mathcal{O}}_j \tilde{\mathcal{O}}^\star_k - \tilde{\mathcal{O}}^\star_j \tilde{\mathcal{O}}_k\right)|0\rangle, \tag{2.4}$$

where the $\star$-conjugation changes left-moving oscillators into right-moving ones and vice-versa without changing their order. The expression (2.4) is invariant under world-sheet parity $\mathcal{P}$ defined by

$\mathcal{P}\Phi = -\Phi^\star$; it is easy to see from an argument similar to the one in [5], that we can consistently restrict the string field to have $\mathcal{P}$-eigenvalue one. The open fields belong to the Hilbert space

$$\tilde{\mathcal{H}}_{\text{open}} = \text{Span}\left\{\alpha^X_{-i_1}\dots\alpha^X_{-i_p} L'_{-j_1}\dots L'_{-j_q} b_{-k_1}\dots b_{-k_r} c_{-\ell_1}\dots c_{-\ell_s} c_1|0\rangle\right\}, \tag{2.5}$$

where

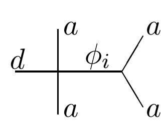
$$i_1 \geq i_2 \geq \dots i_p \geq 1, \quad j_1 \geq j_2 \geq \dots j_q \geq 2, \quad k_1 \geq \dots k_r \geq 1, \quad \ell_1 \geq \dots \ell_s \geq 1, \tag{2.6}$$

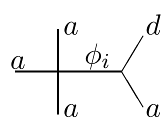
and the L'_{-j} are matter Virasoro operators in the 25-dimensional space orthogonal to X. We can further restrict the closed string field by noting that in the diagrams (2.3) and all other diagrams in this section, the components $|\Phi_i\rangle$ must couple via a cubic vertex to n marginal fields and $3-n$ dilatons. These couplings are zero unless the numbers of α's and the number of $\bar{\alpha}$'s in $|\Phi_i\rangle$ have the same parity which must be opposite to the parity of the ghost numbers of the open fields composing $|\Phi_i\rangle$. Moreover, since a Virasoro L'_{-j} with odd index j can couple only to another Virasoro of odd index, we must have an even number of odd-indexed Virasoro's in $|\Phi_i\rangle$. With the above rules it is straightforward to construct the closed string field needed in this section. At level zero, we have only the tachyon $tc_1\bar{c}_1|0\rangle$, at level two we have the dilaton and marginal field, then at levels 4, 6, 8, 10 and 12 (the highest level considered in this paper) we have respectively 7, 11, 92, 188 and 1016 fields.

We can now continue the calculation of the effective term $\kappa^2 V^{\text{eff}}_{a^4d}$. First we separate the amplitude (2.3) into a contact term and a Feynman term

$$\kappa^2 V^{\text{eff}}_{a^4d} = \kappa^2 V_{a^4d} + \mathcal{C}_{a^4d}, \tag{2.7}$$

and we will focus on the Feynman term

$$-\,4!\,i\,\mathcal{C}_{a^4d} = \sum_i$$


$$+\sum_i$$


. (2.8)

Since at each level greater than zero we have several scalar fields, which are in general not normalized, the propagators in (2.8) will be nondiagonal matrices. We emphasize that the sums in (2.8) would be really meaningful only if the fields were orthogonal, but in our case they must be understood schematically although their meaning remains clear. It will be very convenient to express each of the Feynman diagram in terms of matrix multiplication. We introduce the following notations. $\tilde{A}_{\phi_i\phi_j}$ and $\tilde{A}_{\phi_i\phi_j\phi_k}$ are vectors[1], whose components are given by the coupling constants

$$\begin{aligned}\left(\tilde{A}_{\phi_i\phi_j}\right)_k &\equiv \{\Phi_i, \Phi_j, \Phi_k\} \\ \left(\tilde{A}_{\phi_i\phi_j\phi_k}\right)_h &\equiv \{\Phi_i, \Phi_j, \Phi_k, \Phi_h\}\,,\end{aligned} \tag{2.9}$$

and $\tilde{P}$ is the zero-momentum propagator, a matrix given by

$$\tilde{P} = -\tilde{M}^{-1} \quad \text{where} \quad \tilde{M}_{ij} = \langle\Phi_i|c_0^- Q_B|\Phi_j\rangle. \tag{2.10}$$

[1] We reserve the untilded symbols for the universal Hilbert space when we calculate the tachyon and dilaton effective potential in Section 4.

We can now simply translate (2.8) into

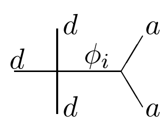
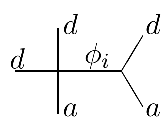
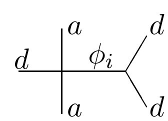
$$-\,4!\,i\,\mathcal{C}_{a^4d}=-6\,i\,\tilde{A}^T_{a^2d}\tilde{P}\tilde{A}_{a^2}-4\,i\,\tilde{A}^T_{a^3}\tilde{P}\tilde{A}_{ad},\tag{2.11}$$

where the only nontriviality is to write the combinatorial weights of each diagram. Note that the factors $(-i)$ in the right-hand side come from two vertices ($(-i)^2$) and one propagator (i). We thus have

$$\mathcal{C}_{a^4d}=\frac{1}{4}\tilde{A}^T_{a^2d}\tilde{P}\tilde{A}_{a^2}+\frac{1}{6}\tilde{A}^T_{a^3}\tilde{P}\tilde{A}_{ad}.\tag{2.12}$$

We emphasize that the expression (2.12) is exact in the infinite level limit, where all the vectors $\tilde{A}$ and the matrix $\tilde{P}$ have infinite size. In the level truncation we restrict the internal fields ϕ_i in the propagators to have level not greater than, say ℓ. And we define $\mathcal{C}_{a^4d}(\ell)$ by the expression (2.12) where the matrix $\tilde{P}$ and vectors $\tilde{A}$ are truncated to finite size, including only the indices related to fields of level smaller than or equal to ℓ. The same convention will apply to all other amplitudes $\mathcal{C}(\ell)$ in this paper. For the way to compute the quartic terms $\tilde{A}_{\phi_i\phi_j\phi_k}$ we refer the reader to [6, 10, 11, 8]. We have computed them here up to level twelve, the values of $\mathcal{C}_{a^4d}(\ell)$ are shown in Table 1.

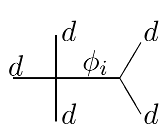
The computation of $\mathcal{C}_{a^2d^3}$ is done in the same way. This time we have three diagrams

$-\,12i\,\mathcal{C}_{a^2d^3}=\sum_i$  $+\sum_i$  $+\sum_i$  , (2.13)

from which we can write

$$\mathcal{C}_{a^2d^3}=\frac{1}{12}\tilde{A}^T_{d^3}\tilde{P}\tilde{A}_{a^2}+\frac{1}{2}\tilde{A}^T_{ad^2}\tilde{P}\tilde{A}_{ad}+\frac{1}{4}\tilde{A}^T_{a^2d}\tilde{P}\tilde{A}_{d^2}.\tag{2.14}$$

And we present the values $\mathcal{C}_{a^2d^3}(\ell)$ in Table 1. For completeness we also compute $\mathcal{C}_{d^5}$. This amplitude was already computed in [9] to level ten and already seen to convincingly cancel the contact term, but we want to extend it here to level twelve so that the calculation is complete, and also so that we have more data to test the fits in Section 3. Here there is only one diagram, namely

$-\,5!\,i\,\mathcal{C}_{d^5}=\sum_i$  $\Rightarrow\quad \mathcal{C}_{d^5}=\frac{1}{12}\tilde{A}^T_{d^3}\tilde{P}\tilde{A}_{d^2}.$ (2.15)

And we list the values of $\mathcal{C}_{d^5}(\ell)$ in Table 1. We also write in this table the extrapolated values $\mathcal{C}(\infty)$ calculated from the fit (3.6) which will be explained in Section 3. And in the last line we show the contact terms calculated with the program described in [9]. We relegate the technical details of the contact terms calculations to Appendix A. We see from Table 1, that the contact terms cancel the contributions from the Feynman diagrams with an accuracy well within the error margins on the contact terms. This is good evidence that, as we expected, the effective potential of the exactly marginal field a and the dilaton d, is flat. It also shows that the quintic contact terms of two different kinds of fields, are computed correctly. In fact the accuracy of the cancellation even suggests that the error on the quintic terms has been overestimated. This possibility was already discussed in [9], but at present this is still the best error estimates that we can do.

	$\mathcal{C}_{a^4d}(\ell)$	$\mathcal{C}_{a^2d^3}(\ell)$	$\mathcal{C}_{d^5}(\ell)$
$\ell = 0$	2.09955	-1.85370	0.401963
$\ell = 4$	1.43546	-1.65253	0.362003
$\ell = 6$	1.42224	-1.50815	0.325946
$\ell = 8$	1.38644	-1.47248	0.316744
$\ell = 10$	1.38545	-1.45971	0.311198
$\ell = 12$	1.38004	-1.45361	0.309417
$\ell = \infty$	1.3774	-1.4457	0.3063
contact term	-1.3779 ± 0.0024	1.4452 ± 0.0053	-0.3063 ± 0.0016

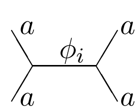
Table 1: The marginal amplitudes from Feynman diagrams with internal fields up to level twelve, and their extrapolations from the fit (3.6). In the last line we list the contact terms whose computations are explained in Appendix A.

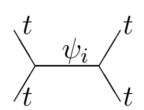
3 Level truncation fits

In this section, we want to find and motivate a good functional form for a fit of closed string amplitudes $\mathcal{C}(\ell)$ as functions of the level ℓ. We start by remembering that in open string field theory, computations to very high levels (typically 100) have been done (see for example [12]) and it turns out that fits of the form

$$\mathcal{C}^{\text{fit}}_{\text{open}}(\ell) = f_0 + \frac{f_1}{\ell} + \frac{f_2}{\ell^2} + \frac{f_3}{\ell^3} + \ldots + \frac{f_N}{\ell^N}$$

perform very well. We emphasize that the next-to-leading term is of order ℓ^{-1}, as was shown from the BST algorithm [13]. Some particular closed string field theory amplitudes, like



or


,

where the propagating fields ϕ_i and ψ_i are tensor products of twist-even open fields of ghost number one, can be expressed in terms of open string amplitudes. In these cases it was shown in [10] that the next-to-leading order of the fit is ℓ^{-2}. One might then suggest that closed string amplitudes should be fitted with

$$\mathcal{C}^{\text{fit}}(\ell) = f_0 + \frac{f_2}{\ell^2} + \frac{f_3}{\ell^3} + \ldots + \frac{f_N}{\ell^N}. \tag{3.1}$$

But it was found [11] that this fit doesn't perform well for amplitudes that cannot be expressed in terms of open physical amplitudes. Instead, fits of the form

$$\mathcal{C}^{\text{fit}}(\ell) = f_0 + \frac{f_1}{\ell^\gamma} \tag{3.2}$$

seem to work better once the exponent γ has been adjusted in some way. In particular the authors of [11] found that $\gamma = 2.7$ and $\gamma = 3.2$ for the fits of $\mathcal{C}_{a^2d^2}$ and $\mathcal{C}_{a^4}$ respectively give the expected values as $\ell \to \infty$ (the ones that cancel the quartic contact terms).

One could go on and imagine many variants of the above fits, for example by adding a term $\frac{f_2}{\ell^{2\gamma}}$ to (3.2) etc... In order to argue what fits are better, we must take a look at Table 1. The first thing that we emphasize is that we will keep only the data points $\ell = 4$, $\ell = 8$ and $\ell = 12$. Indeed we see for example in the first column of the table, that the values for $\ell = 4$ and $\ell = 6$ are very similar, as well as the values for $\ell = 8$ and $\ell = 10$. This is easy to understand. Fields of level $4n+2$ are made of open fields of odd level $2n+1$; but in open string field theory, the parity of level is very important, indeed the twist symmetry implies that the open vertex can couple only an even number of odd level fields of ghost number one (this is why one can consistently set these fields to zero in the nonperturbative open string vacuum for example). So the similarities between levels $4n$ and $4n+2$ are just remnants of twist symmetry. Were we to plot $\mathcal{C}(\ell)$ for all values of ℓ, we would obtain a rather stair-looking curve, while if we keep only levels $4n$ (or $4n+2$) the curve is smoother and thus easier to fit. At last we throw away the value at $\ell = 0$ as the fits are singular there[2].

The second observation that we can make on Table 1, is that the values of $\mathcal{C}(\ell)$ behave *monotonically* with the level ℓ. We will assume that this monotonicity is a feature of all amplitudes and persists at high level. For definiteness, let us now consider a $\mathcal{C}(\ell)$ which is monotonically decreasing. This monotonicity imposes strong restrictions on a good fit of $\mathcal{C}(\ell)$ because we want the value of the fit at $\ell \to \infty$ to be better, i.e. *smaller*, than the last data point. If the number of data points that we are fitting is greater than the number of parameters in our fit, the fit will not go exactly through the data points, and there is an unacceptable risk that the fit at infinity will give a value larger than our best data point. There are other restrictions; indeed, if we take the fit (3.1) and all three of our data points, keeping thus three fit parameters f_0, f_2 and f_3, it might happen that f_2 and f_3 have different signs, which would imply that the fit is not monotonically decreasing and we might again end up with a fitted value at infinity worse than the best data point. We will therefore choose a fit of the form (3.2).

But we experienced that if we use the three data points at $\ell = 4$, $\ell = 8$ and $\ell = 12$ to set f_0, f_1 and γ, the fits are sometimes quite poor in the sense that the value of the fit at $\ell \to \infty$ does not satisfactorily cancel the quintic contact term. But in those cases, we also observed that the value of γ chosen by the fit, is far away from 3. Let us then try to set $\gamma = 3$ from the beginning

$$\mathcal{C}^{\text{fit}}(\ell) = f_0 + \frac{f_1}{\ell^3} \tag{3.3}$$

and use the data points at $\ell = 8$ and $\ell = 12$ to determine f_0 and f_1, we have then explicitly

$$\mathcal{C}^{\text{fit}}(\infty) = f_0 = \frac{1}{19}\left(27\,\mathcal{C}(12) - 8\,\mathcal{C}(8)\right). \tag{3.4}$$

The values from this fit for the marginal amplitudes of Section 2, are shown in Table 1; they cancel the contact terms with a striking precision. The fit (3.3) therefore seems to be excellent, except

[2] One could of course fix that singular behavior by, for example, replacing ℓ by $\ell + \ell_0$ in (3.1) or (3.2), but we observed that the resulting fits are not improved.

for the amplitudes mentioned at the beginning of this section, those whose internal (propagating) fields are tensor products of physical (i.e. ghost number one) twist-even open fields, whose fit we know should rather be

$$\mathcal{C}^{\rm fit}(\ell) = f_0 + \frac{f_1}{\ell^2}. \tag{3.5}$$

All in all, we conclude that a good fit of closed amplitudes $\mathcal{C}(\ell)$, is (3.5) if the internal fields are tensor products of open physical and twist-even fields, and (3.3) otherwise, and that we should keep only the maximum available levels L and $L-4$ in order to determine f_0 and f_1. We can thus express $\mathcal{C}^{\rm fit}(\infty) = f_0$ explicitly in terms of $\mathcal{C}(L)$ and $\mathcal{C}(L-4)$, namely

$$\boxed{\begin{array}{l} \mathcal{C}^{\rm fit}(\infty) = \dfrac{L^\gamma\, \mathcal{C}(L) - (L-4)^\gamma\, \mathcal{C}(L-4)}{L^\gamma - (L-4)^\gamma}, \\ \text{where } \gamma = \begin{cases} 2 & \text{if internal fields are} \otimes \text{ of open phys. twist-even fields} \\ 3 & \text{otherwise} \end{cases} \end{array}} \tag{3.6}$$

In order to test further the fit (3.6) we redo, to level twelve, the calculation of quartic marginal deformations that was done in [10, 11]. The results are shown in Table 2. The fit projections for

	$\mathcal{C}_{a^4}(\ell)$	$\mathcal{C}_{a^2d^2}(\ell)$	$\mathcal{C}_{d^4}(\ell)$
$\ell = 8$	0.265827	-0.483015	0.115777
$\ell = 10$	0.265827	-0.469970	0.108550
$\ell = 12$	0.259977	-0.465334	0.108499
$\ell = \infty$	0.2553	-0.4579	0.1054
contact term	-0.2560	0.4571	-0.1056

Table 2: The quartic marginal amplitudes from Feynman diagrams at levels 8, 10 and 12, and their extrapolations from the fit (3.6). The last line shows the contact terms.

$\mathcal{C}_{a^2d^2}$ and $\mathcal{C}_{d^4}$, cancel the contact terms with substantially more accuracy than the fits [11] from level six data. This is especially interesting in the case of $\mathcal{C}_{a^2d^2}$; had we fitted it with (3.2) and $\gamma = 5/2$ as was done in [11], we would have found $\mathcal{C}^{\rm fit}_{a^2d^2}(\infty) = -0.4553$, a worse result than what we find with $\gamma = 3$. The fit of $\mathcal{C}_{a^4}$ is however a little poorer here than in [10] (where the projection was 0.2559). Note that the propagator of this amplitude only involves fields which are tensor products of open physical twist-even fields (this can also be seen from the fact that the values at levels 8 and 10 are the same), and we should therefore take $\gamma = 2$. The fact that the data to level six gives a better answer than the data to level twelve with the same functional form of fit (with $\gamma = 2$) is probably accidental. Anyway, had we used $\gamma = 3$ we would have found $\mathcal{C}^{\rm fit}_{a^4}(\infty) = 0.2575$, not as good as with $\gamma = 2$. This is thus good evidence that the choice of γ in (3.6) is right.

4 The effective potential

We are now ready to confidently calculate the effective tachyon and dilaton potential to order five. Indeed we have shown that we can trust the quintic contact terms computations needed, and we have a good fit at hand to extrapolate the results to infinite level. We start with the order four (where quintic computations are not needed), which had already been calculated in [5] to level four, but we are going to level twelve and extrapolating; we will see that to this order, the effective potential provides unexpectedly accurate results for the Yang-Zwiebach vacuum [5]. We will then proceed to order five and discuss the local extrema of the potential.

We start by giving here a few definitions. The closed string field $|\Psi\rangle = \sum_i \psi_i|\Psi_i\rangle$ is in the universal Hilbert space, and is as described in [5, 8]. We again split contact term and Feynman contribution

$$\kappa^2 V^{\mathrm{eff}}_{\psi_1\psi_2\ldots\psi_N} = \kappa^2 V_{\psi_1\psi_2\ldots\psi_N} + \mathcal{C}_{\psi_1\psi_2\ldots\psi_N}. \tag{4.1}$$

And we use the following notations; $A_{\psi_i\psi_j}$ and $A_{\psi_i\psi_j\psi_k}$ are vectors, whose components are given by

$$\begin{aligned}\left(A_{\psi_i\psi_j}\right)_k &\equiv \{\Psi_i, \Psi_j, \Psi_k\}\\ \left(A_{\psi_i\psi_j\psi_k}\right)_h &\equiv \{\Psi_i, \Psi_j, \Psi_k, \Psi_h\}\,,\end{aligned} \tag{4.2}$$

and B_{ψ_i} are matrices with components

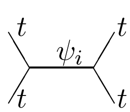
$$\left(B_{\psi_i}\right)_{jk} \equiv \{\Psi_i, \Psi_j, \Psi_k\}\,. \tag{4.3}$$

Since the multilinear string functions are totally symmetric, B_{ψ_i} are symmetric matrices; and it doesn't matter in which order the index fields of A are written. At last P is the zero-momentum propagator, a matrix given by

$$P = -M^{-1} \quad \text{where} \quad M_{ij} = \langle\Psi_i|c_0^- Q_B|\Psi_j\rangle. \tag{4.4}$$

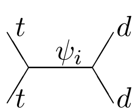
4.1 Order four

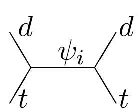
We calculate here the terms $\kappa^2 V^{\mathrm{eff}}_{d^n t^{4-n}}$ for $n = 0, \ldots, 4$. The manipulations are similar to those of Section 2. Since the Feynman diagrams involve only cubic vertices, only those with an even number of dilaton can be nonzero. For $\mathcal{C}_{t^4}$ we find

$$\mathcal{C}_{t^4} = \frac{i}{4!}\sum_i$$


$$= \frac{1}{8}A_{tt}^T P A_{tt}, \tag{4.5}$$

where the internal fields ψ_i are all the scalars except the tachyon and dilaton. And for $\mathcal{C}_{t^2d^2}$ we have

$$\mathcal{C}_{t^2d^2} = \frac{i}{4}\left(\sum_i\right.$$


$$+\sum_i$$


$$\left.\vphantom{\sum_i}\right) = \frac{1}{4}A_{tt}^T P A_{dd} + \frac{1}{2}A_{td}^T P A_{td}. \tag{4.6}$$

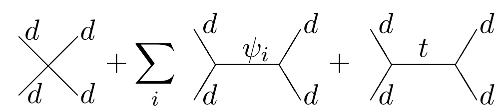
The results to level twelve and their extrapolations are shown in Table 3. The Feynman contribution

ℓ	$\mathcal{C}_{t^4}(\ell)$	$\mathcal{C}_{t^2d^2}(\ell)$
4	$-\frac{1896129}{4194304} \approx -0.452072$	$\frac{25329}{16384} \approx 1.54596$
6	$-\frac{1896129}{4194304} \approx -0.452072$	$\frac{19104841}{11943936} \approx 1.59954$
8	$-\frac{24710749}{50331648} \approx -0.490958$	$\frac{178516846189}{104485552128} \approx 1.70853$
10	$-\frac{24710749}{50331648} \approx -0.490958$	$\frac{179239681645}{104485552128} \approx 1.71545$
12	$-\frac{16280361760337731}{32499186133893120} \approx -0.500947$	$\frac{17898902809317331}{10282945612677120} \approx 1.74064$
∞	-0.5089	1.754

Table 3: The Feynman contributions needed for the computation of the effective potential at order four.

for the term $\kappa^2 V^{\text{eff}}_{d^4}$ is not needed because we can use the dilaton theorem

$$0 =$$



$$= -4!\, i\, \kappa^2 V^{\text{eff}}_{d^4} - 3\, i\, \{D, D, T\} \left(\frac{1}{2}\right) \{T, D, D\}\,, \quad (4.7)$$

from which we deduce

$$\kappa^2 V^{\text{eff}}_{d^4} = -\frac{1}{16}\{D, D, T\}^2 = -\frac{729}{4096} \approx -0.1780. \quad (4.8)$$

We now just need the contact terms (see [5] for example)

$$\kappa^2 V_{t^4} = -3.017, \quad \kappa^2 V_{t^3 d} = 3.872, \quad \kappa^2 V_{t^2 d^2} = 1.368, \quad \kappa^2 V_{t d^3} = -0.9528. \quad (4.9)$$

All in all we have for the potential at order four

$$\kappa^2 V^{\text{eff}}_4 = -t^2 + \frac{6561}{4096}\, t^3 - \frac{27}{32}\, t d^2 - 3.526\, t^4 + 3.872\, t^3 d + 3.122\, t^2 d^2 - 0.9528\, t d^3 - \frac{729}{4096}\, d^4. \quad (4.10)$$

In order to judge how well it captures the vacuum structure, we will compare the results for the local extremum found in truncation scheme B of [8] and the analog found with the effective potential truncated to fields of level L, with $L = 4, 6, 8, 10$. The analogs of (4.10) with internal fields of levels not greater than L are

$$\begin{aligned}
\kappa^2 V^{\text{eff}}_{4,4} &= -\,t^2 + \frac{6561}{4096}\, t^3 - \frac{27}{32}\, t d^2 - 3.469\, t^4 + 3.872\, t^3 d + 2.914\, t^2 d^2 - 0.9528\, t d^3 - 0.1390\, d^4 \\
\kappa^2 V^{\text{eff}}_{4,6} &= -\,t^2 + \frac{6561}{4096}\, t^3 - \frac{27}{32}\, t d^2 - 3.469\, t^4 + 3.872\, t^3 d + 2.968\, t^2 d^2 - 0.9528\, t d^3 - 0.1673\, d^4 \\
\kappa^2 V^{\text{eff}}_{4,8} &= -\,t^2 + \frac{6561}{4096}\, t^3 - \frac{27}{32}\, t d^2 - 3.508\, t^4 + 3.872\, t^3 d + 3.077\, t^2 d^2 - 0.9528\, t d^3 - 0.1678\, d^4 \\
\kappa^2 V^{\text{eff}}_{4,10} &= -\,t^2 + \frac{6561}{4096}\, t^3 - \frac{27}{32}\, t d^2 - 3.508\, t^4 + 3.872\, t^3 d + 3.083\, t^2 d^2 - 0.9528\, t d^3 - 0.1750\, d^4
\end{aligned} \quad (4.11)$$

We show in Table 4, the value of the potential for the vacuum found in truncation scheme B [8] at fields level L, compared to the values of the extrema of the potentials (4.11). We emphasize

L	4	6	8	10	∞
value of $\kappa^2 V^{\text{eff}}_{4,L}$	-0.05443	-0.05415	-0.05266	-0.05274	-0.05234
value of $\kappa^2 V_{L,4L}$ in scheme B	-0.05442	-0.0544	-0.0514	-0.0513	-0.050

Table 4: Comparison of the values of the effective potential and the full potential at the nonperturbative vacuum of [5, 8]. The last line was calculated in Section 3 of [8].

that only the value at $L = 4$ of $\kappa^2 V_{L,4L}$ in truncation scheme B, is exact. The other ones were obtained by extrapolating the values of $\kappa^2 V_{L,M}$ to $M = 4L$. And the value at infinity was in turn extrapolated from the values of the last line of Table 4. We see a striking similarity between the values at fields level $L = 4$ (the small mismatch is within the relative expected error made on the quartic terms, which is about 0.1%). Could these values be exactly equal (and the mismatch of the others be due to extrapolation errors)? We shouldn't expect so. Indeed if we wanted to calculate the effective potential from the potential, by solving the equations of motion for all the massive fields for fixed values of t and d, and plug back into the potential the resulting expressions of the massive fields as functions of t and d, we should obtain a nonpolynomial function of t and d. This function would agree with $\kappa^2 V^{\text{eff}}_{4,4}$ to order four, but we will have terms of higher order as well. Those will lack the contact terms of course, but they will contain terms from Feynman diagrams built with cubic and quartic vertices. It is instructive to compare the tachyon and dilaton vacuum expectation values. From the effective potential $V^{\text{eff}}_{4,4}$ we find

$$(t, d) = (0.3424, 0.4057), \tag{4.12}$$

while from $V_{4,16}$ in scheme B we find

$$(t, d) = (0.3265, 0.4349). \tag{4.13}$$

This rules out strict equality, but these two results are not that different. We will thus interpret the numerical values in Table 4, as evidence for the following approximate conjecture.

Conjecture 1 *The effective tachyon and dilaton potential $\kappa^2 V^{\text{eff}}_N$ to a given polynomial order N, captures with good approximation the physics of the whole potential including vertices up to order N and with* all *interactions from the untruncated string field.*

We emphasize that this is not a precise statement as we are only stating an approximation. This is nevertheless a strong statement; it implies in particular that at order five, we may only calculate the contact terms $\kappa^2 V_{t^5}$, $\kappa^2 V_{t^4 d}$, $\kappa^2 V_{t^3 d^2}$, $\kappa^2 V_{t^2 d^3}$, $\kappa^2 V_{t d^4}$ and $\kappa^2 V_{d^5}$ necessary to form the effective potential, and that we will have a good approximation of the vacuum structure of the potential with all quintic contact terms (to fields level four there are 252 such terms, to level six there are 20,349 of them! And then we would still need to extrapolate to infinite level).

Before going to order five, we want to do one more thing at order four. We want to find all extrema of the potential (4.10) and check whether they are local maxima, minima, or saddle points. In order to do this we will look at the eigenvalues λ_1 and λ_2 of the matrix S of second derivatives

$$S = \kappa^2 \begin{pmatrix} \partial_t^2 V^{\text{eff}} & \partial_t \partial_d V^{\text{eff}} \\ \partial_d \partial_t V^{\text{eff}} & \partial_d^2 V^{\text{eff}} \end{pmatrix}. \tag{4.14}$$

Keeping only the real nontrivial solutions (and throwing away those which are very close to the origin and merely artifacts of truncation) we find three extrema. The one corresponding to the Yang-Zwiebach vacuum is

$$(t, d) = (0.3348, 0.4005), \quad \kappa^2 V_4^{\text{eff}} = -0.05234, \quad (\lambda_1, \lambda_2) = (-2.192, 1.810). \tag{4.15}$$

We have one negative and one positive eigenvalue, this vacuum is therefore a saddle point. This is interesting, it means that it cannot be a true vacuum of the theory. In other words, the theory expanded at this vacuum still has a tachyon (of mass squared λ_1). What about the other two vacua? We have one vacuum with a negative dilaton vev

$$(t, d) = (0.2497, -0.8229), \quad \kappa^2 V_4^{\text{eff}} = -0.06062, \quad (\lambda_1, \lambda_2) = (-4.236, 1.148), \tag{4.16}$$

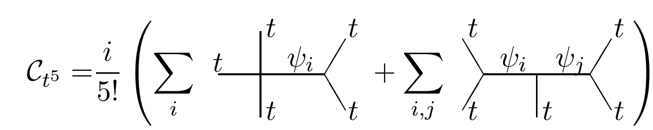
which is again a saddle point. The third vacuum has a negative tachyon vev

$$(t, d) = (-0.1312, -0.4829), \quad \kappa^2 V_4^{\text{eff}} = -0.003062, \quad (\lambda_1, \lambda_2) = (-1.967, 0.3736), \tag{4.17}$$

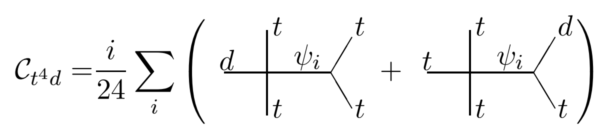
again a saddle point. But we notice that t and λ_2 are rather small, we interpret this as this point belonging to the family of vacua generated by the dilaton deformations of the perturbative vacuum; it is an artifact of truncation that we find only a finite number of these vacua.

4.2 Order five

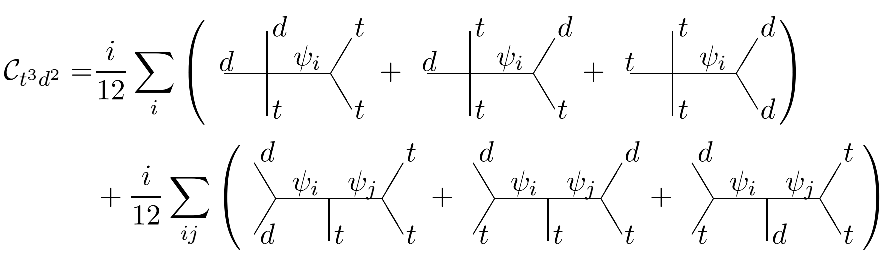
We now compute the effective potential to order five, and in the light of the last section we hope that it may give us a good insight into the vacuum structure of the theory. We start by calculating the Feynman contributions



$$= \frac{1}{12} A_{ttt}^T P A_{tt} + \frac{1}{8} A_{tt}^T P B_t P A_{tt} \tag{4.18}$$

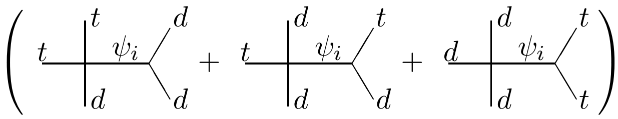


$$= \frac{1}{4} A_{ttd}^T P A_{tt} + \frac{1}{6} A_{ttt}^T P A_{td} \tag{4.19}$$



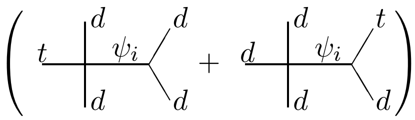
$$
\begin{aligned}
&= \frac{1}{4} A_{tdd}^T P A_{tt} + \frac{1}{2} A_{ttd}^T P A_{td} + \frac{1}{12} A_{ttt}^T P A_{dd} \\
&\quad + \frac{1}{4} A_{dd}^T P B_t P A_{tt} + \frac{1}{2} A_{td}^T P B_t P A_{td} + \frac{1}{2} A_{td}^T P B_d P A_{tt}
\end{aligned}
\tag{4.20}
$$

$$\mathcal{C}_{t^2d^3} = \frac{i}{12} \sum_i$$

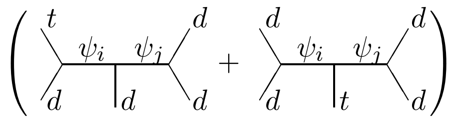


$$= \frac{1}{4} A_{ttd}^T P A_{dd} + \frac{1}{2} A_{tdd}^T P A_{td} + \frac{1}{12} A_{ddd}^T P A_{tt} \tag{4.21}$$

$$\mathcal{C}_{td^4} = \frac{i}{24} \sum_i$$



$$+ \frac{i}{24} \sum_{ij}$$



$$= \frac{1}{4} A_{tdd}^T P A_{dd} + \frac{1}{6} A_{ddd}^T P A_{td} + \frac{1}{2} A_{td}^T P B_d P A_{dd} + \frac{1}{8} A_{dd}^T P B_t P A_{dd}. \tag{4.22}$$

The results are shown in Table 5. The corresponding contact terms are computed with the

ℓ	$\mathcal{C}_{t^5}(\ell)$	$\mathcal{C}_{t^4d}(\ell)$	$\mathcal{C}_{t^3d^2}(\ell)$	$\mathcal{C}_{t^2d^3}(\ell)$	$\mathcal{C}_{td^4}(\ell)$
4	3.79575	−1.55833	−7.51218	3.17206	1.05369
6	3.79575	−1.61549	−8.15761	3.41664	1.33655
8	4.17801	−1.73714	−8.80564	3.59308	1.54958
10	4.17801	−1.74333	−8.89440	3.61552	1.62033
12	4.27270	−1.77456	−9.03854	3.65374	1.66610
∞	4.348	−1.790	−9.137	3.679	1.715

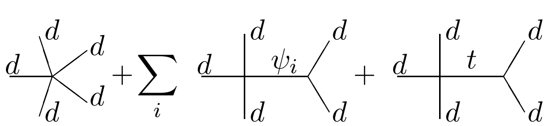
Table 5: The Feynman contributions to the order five of the effective potential, and their extrapolations to infinite level using the fit (3.6).

$\kappa^2 V_{t^5}$	$\kappa^2 V_{t^4d}$	$\kappa^2 V_{t^3d^2}$	$\kappa^2 V_{t^2d^3}$	$\kappa^2 V_{td^4}$
9.924 ± 0.008	-20.613 ± 0.026	4.702 ± 0.021	6.769 ± 0.021	-0.8077 ± 0.0036

Table 6: The quintic contact terms needed at the order five of the effective potential. Details on their computation can be found in Appendix A.

program described in [9], and shown in Table 6. The details are explained in Appendix A. For the term $\kappa^2 V_{d^5}^{\rm eff}$ we can again use the dilaton theorem to write

$$0 =$$



$$= -5!\, i\, \kappa^2 V_{d^5}^{\rm eff} - 10\, i\ \{D,D,D,T\} \left(\frac{1}{2}\right) \{T,D,D\}\,, \tag{4.23}$$

and thus

$$\kappa^2 V^{\rm eff}_{d^5} = -\frac{1}{24}\{D,D,D,T\}\{T,D,D\} = -0.4020. \tag{4.24}$$

And finally we can write down the effective potential at order five

$$\boxed{\begin{aligned}\kappa^2 V^{\rm eff}_5 = -t^2 + \frac{6561}{4096}\,t^3 - \frac{27}{32}\,td^2 - 3.526\,t^4 + 3.872\,t^3 d + 3.122\,t^2 d^2 - 0.9528\,td^3 - \frac{729}{4096}\,d^4 \\ +14.27\,t^5 - 22.40\,t^4 d - 4.435\,t^3 d^2 + 10.45\,t^2 d^3 + 0.9073\,td^4 - 0.4020\,d^5\end{aligned}}\,. \tag{4.25}$$

We can now do the same vacuum search as we did to order four. This time we find five real nontrivial extrema. The one corresponding to the Yang-Zwiebach vacuum is

$$\boxed{(t,d) = (0.2105, 0.4582), \quad \kappa^2 V^{\rm eff}_5 = -0.03322, \quad (\lambda_1,\lambda_2) = (-2.311, 1.870)}\,. \tag{4.26}$$

In addition to this one, we find three other saddle points

$$\begin{aligned}
&(t,d) = (0.2676, -0.1185), && \kappa^2 V^{\rm eff}_5 = -0.03662, && (\lambda_1,\lambda_2) = (-0.5878, 4.594)\\
&(t,d) = (0.9881, 0.8575), && \kappa^2 V^{\rm eff}_5 = 0.06579, && (\lambda_1,\lambda_2) = (-3.112, 82.48)\\
&(t,d) = (-0.4221, -0.5721), && \kappa^2 V^{\rm eff}_5 = -0.07998, && (\lambda_1,\lambda_2) = (-9.067, 2.848).
\end{aligned} \tag{4.27}$$

But we now have a *minimum*

$$\boxed{(t,d) = (0.4907, 0.3978), \quad \kappa^2 V^{\rm eff}_5 = -0.08245, \quad (\lambda_1,\lambda_2) = (0.9509, 8.841)}\,. \tag{4.28}$$

Before we discuss these results in Section 6, we try the usual level truncation scheme in the next section.

5 Usual level truncation

In this section we want to address the question of tachyon condensation in the level truncation by looking for extrema of the potential itself (not the effective potential). There are two main approaches to level truncation, which were denoted schemes A and B respectively in [8]. Here, the analog of scheme A would be to expand the string field to a large given level and include as many cubic and quartic interactions as possible, we would then include quintic interactions level by level. In scheme B, we would increase the level of the string field step by step, and include *all* the cubic, quartic and quintic interactions. In [8] it was seen that convergence is better in scheme B, but the computations of all quartic interactions was a challenge that could be completely achieved only to string field level four. Here the quintic term is, of course, even more challenging. At level two, the result is essentially included in the effective potential discussed in Section 4. At level four, we would need to include all quintic terms up to total level twenty (a total of 252 terms); this is beyond the scope of this work. We will therefore focus on scheme A in this section.

We will truncate the string field to level four, namely

$$\begin{aligned}|\Psi\rangle = t\, c_1\bar c_1|0\rangle + d\,(c_1 c_{-1} - \bar c_1 \bar c_{-1})|0\rangle + f_1\, c_{-1}\bar c_{-1}|0\rangle + f_2\, L_{-2}c_1\bar L_{-2}\bar c_1|0\rangle \\ + f_3\,(L_{-2}c_1\bar c_{-1} - \bar L_{-2}\bar c_1 c_{-1})|0\rangle + g_1\,(b_{-2}c_1\bar c_{-2}\bar c_1 - \bar b_{-2}\bar c_1 c_{-2} c_1)|0\rangle,\end{aligned} \tag{5.1}$$

and we will include all the cubic and quartic interactions, and the quintic interactions at levels zero, two and four. We will therefore need the quintic contact terms $\kappa_2 V_{t^5}$, $\kappa_2 V_{t^4 d}$ and $\kappa_2 V_{t^3 d^2}$ (see Table 6) and the terms $\kappa_2 V_{t^4 f_1}$, $\kappa_2 V_{t^4 f_2}$, $\kappa_2 V_{t^4 f_3}$ and $\kappa_2 V_{t^4 g_1}$ shown in Table 7. The details of these

$\kappa^2 V_{t^4 f_1}$	$\kappa^2 V_{t^4 f_2}$	$\kappa^2 V_{t^4 f_3}$	$\kappa^2 V_{t^4 g_1}$
0.4059 ± 0.0046	244.98 ± 0.48	-50.43 ± 0.10	-3.9353 ± 0.0068

Table 7: The contact terms of four tachyons and one field of level four.

computations can be found in Appendix A. The quintic potentials at each level are thus

$$\begin{aligned}
\kappa^2 V_0^{(5)} &= 9.924\, t^5 \\
\kappa^2 V_2^{(5)} &= -20.61\, t^4 d \\
\kappa^2 V_4^{(5)} &= 4.702\, t^3 d^2 + t^4 \left(0.4059\, f_1 + 245.0\, f_2 - 50.43\, f_3 - 3.935\, g_1\right).
\end{aligned} \tag{5.2}$$

And the total potentials are

$$\begin{aligned}
\mathbb{V}_0^{(5)} &= \mathbb{V}_{4,16}^{(4)} + V_0^{(5)} \\
\mathbb{V}_2^{(5)} &= \mathbb{V}_0^{(5)} + V_2^{(5)} \\
\mathbb{V}_4^{(5)} &= \mathbb{V}_2^{(5)} + V_4^{(5)},
\end{aligned} \tag{5.3}$$

where $\mathbb{V}_{4,16}^{(4)}$ contains all the quadratic, cubic, and quartic terms of fields of level up to four (and thus contains interactions of level up to sixteen). We now look for a minimum of these potentials corresponding to the Yang-Zwiebach vacuum. In order to do this, we solve numerically the equations with a start value (a seed) corresponding to this vacuum. The results are shown in Table 8. We

Potential	t	d	f_1	f_2	f_3	g_1	Value
$\kappa^2 \mathbb{V}_{4,16}^{(4)}$	0.3265	0.4349	−0.1221	−0.008973	−0.03845	−0.09332	−0.05442
$\kappa^2 \mathbb{V}_0^{(5)}$	0.2600	0.2373	−0.04735	−0.004174	−0.01530	−0.03555	−0.03281
$\kappa^2 \mathbb{V}_2^{(5)}$	0.2423	−0.3718	−0.009011	0.0001399	−0.003029	0.02344	−0.03802
$\kappa^2 \mathbb{V}_4^{(5)}$	0.1588	−0.6072	−0.04073	−0.0005148	−0.01074	0.03996	−0.02629

Table 8: The extremum of the potential found in the level truncation scheme A.

see that this vacuum is destroyed after we include the term of level two $V_2^{(5)}$. Instead, a local extremum is found at a *negative* value of the dilaton. We have done the same calculation with $\mathbb{V}_0^{(5)} = \mathbb{V}_{10,10}^{(4)} + V_0^{(5)}$, i.e. using fields up to level ten and with cubic interactions up to level 24 and quartics interaction up to level ten; and we found qualitatively the same results as in Table 8. So the breakdown of the solution is really due to the quintic terms. We found another extremum to

the potential $\mathbb{V}_4^{(5)}$ of (5.3), namely

$$(t,d) = (-0.2031, -0.5240), \quad \kappa^2 \mathbb{V}_4^{(5)} = -0.01152. \tag{5.4}$$

It is important to note that none of the extrema, (5.4) or the one in Table 8, correspond to any extremum of the effective potential of Section 4.

6 Conclusions and prospects

In this paper we have shown that we are able to correctly compute quintic contact terms when the interacting fields are not all the same. This was shown by verifying, to order five, that the dilaton and one exactly marginal field form a moduli space of marginal deformations. We then used this data to motivate a universal fit which gives very good approximations for all the verifiable amplitudes that we have computed. This fit was then used in the computation of the tachyon and dilaton effective potential. At order four, we noticed that the extrema from this effective potential were very close (more than expected) to the extrema found from the potential with many terms. We phrased this nice apparent property as a conjecture.

Since it is only an approximate statement, we will interpret Conjecture 1 as a statement on *level truncation*. In other words it tells us that when including the vertex of order N, one should first include the terms $\kappa^2 V_{t^n d^{N-n}}$ which will be the most important contributions, and then include all the terms with level four fields, and so on. This is different from usual truncation as, for example, some terms of level $2N$ are included before some terms of level 4. It would be interesting to check such a truncation scheme in a different context, like tachyons on orbifolds (see [14] for example).

It is a little bit surprising that, at order five, the vacua found from the effective potential do not agree with those found in the usual level truncation scheme A. If we do believe Conjecture 1, we shall give more credence to the results from the effective potential. This is especially reasonable since we went only to level four in the usual truncation scheme. We will take this point of view, and not discuss further the results from usual truncation, except to say that it would of course be interesting to include terms of higher levels.

Of all the saddle points found from the effective potential, only one seems physically meaningful. Indeed the solutions (4.27) have no equivalent at order four; and similarly the saddle points (4.16) and (4.17) have no analog at order five. The Yang-Zwiebach vacuum (4.15), however, survives to order five; moreover the eigenvalues λ_i are stable from order four to order five. This is evidence that this vacuum is physical, present in the full untruncated theory. The value of the potential at this vacuum goes from -0.05234 at order four to -0.03322 at order five. This is certainly compatible with the conjecture [5] that it should be zero. On the other hand, one might be concerned by the fact that the vacuum expectation value of the tachyon goes from 0.3348 at order four to 0.2105 at order five. Is this vacuum simply going to converge to a dilaton deformation of the perturbative vacuum to higher order? One of the eigenvalues λ_i should then tend to zero, but this is clearly

not the case, as can be seen from (4.15) and (4.26). We are thus led to claim that this vacuum is physically interesting. As to its interpretation, the shallowness of the potential certainly supports the interpretation from the low-energy effective action [5, 7] that the universe ends in a big crunch there. But the fact that the Yang-Zwiebach vacuum is not a local minimum but a saddle point certainly raises new questions. On the one hand, one could argue that the big crunch interpretation is so drastic that it doesn't matter that we are not on a stable point. It is even tempting to imagine that the remaining instability could bring the system back to its original perturbative vacuum, and that the universe would thus undergo an infinite cycle of big crunches and big bangs, like in cyclic universe models [15]. On the other hand, one might wonder whether the system will ever reach the saddle point. Indeed, even if the system starts rolling approximately towards it, it seems natural that it will eventually turn to the downward direction and miss it.

But in this paper we have found a local minimum as well (4.28), a very interesting result as it suggests the existence of a stable nonperturbative vacuum. This is found only at order five and has no analog at lower order, it is thus hard to say at this point whether this is a physical result or just an effect of truncation. As for its physical interpretation, it is as hard to say. We can nevertheless note that it has a positive tachyon vev - what we naively expect from a vacuum since negative tachyon values correspond to the unbounded side of the potential at cubic order. It has also a positive dilaton vev, corresponding to large string coupling as argued in [7, 5]. Some clue could be given by the second derivatives of the potentials (the eigenvalues λ_1 and λ_2) which should correspond to the mass squared of two particles found in this vacuum. Those are respectively approximately 1 and 9 (in units where $\alpha' = 2$).

There are several directions in which the present work could be continued. In particular, more quintic contact terms could be computed. This could in particular allow to check Conjecture 1, and see if the Yang-Zwiebach vacuum is restored in the usual level truncation after including more terms. If we want to continue the direct search of a nonperturbative vacuum, however, it seems very desirable to be able to make computations at order six. An extension of [9] to the sixtic term, however, would require tremendous work and very strong programming skills. Other approaches should be considered. Progress on the analytical side would be of course extremely important, but a different numerical approach might be the way to go. For example, if we remember that the most complicated part in the contact term computation [9] was the computation of the boundary of the reduced moduli space, a natural suggestion is to integrate over the *whole* moduli space instead. We would thus produce effective terms (which is good if we believe Conjecture 1); but we would encounter divergences as well, coming from the propagator of the zero-momentum dilaton. It would therefore be very interesting to find a way to deal with these divergences (Belopolsky managed to do this at order four [4]).

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A Quintic contact terms

We collect here the technical results needed to compute the quintic contact terms needed in this paper. All the closed string correlators are given explicitly. Their integration over the moduli space was done with the program developed in [9]; for more details the reader should consult this reference.

A.1 Integration over the reduced moduli space

We begin by recalling how to integrate over the reduced moduli space of spheres with five punctures $\mathcal{V}_{0,5}$. It was shown in [9] that this space can be divided into 120 pieces and that the integration can be written as an integration over one single piece $\mathcal{A}_5$. The five-string multilinear function reads

$$\{\Psi_1,\Psi_2,\Psi_3,\Psi_4,\Psi_5\} = \frac{1}{\pi^2}\int_{\mathcal{V}_{0,5}} dx_1 dy_1 dx_2 dy_2 \langle\Sigma|\left(\mathcal{B}\mathcal{B}^\star\right)_1\left(\mathcal{B}\mathcal{B}^\star\right)_2|\Psi_1\rangle|\Psi_2\rangle|\Psi_3\rangle|\Psi_4\rangle|\Psi_5\rangle, \tag{A.1}$$

where the antighost insertions $\left(\mathcal{B}\mathcal{B}^\star\right)_i$ are given by

$$\mathcal{B}_i = \sum_{I=1}^{5}\sum_{m=-1}^{\infty}\left(B^I_{i,m}b^{(I)}_m + \overline{C^I_{i,m}}\bar{b}^{(I)}_m\right), \quad \mathcal{B}^\star_i = \sum_{I=1}^{5}\sum_{m=-1}^{\infty}\left(C^I_{i,m}b^{(I)}_m + \overline{B^I_{i,m}}\bar{b}^{(I)}_m\right) \tag{A.2}$$

$$B^{(I)}_{i,m} = \oint\frac{dw}{2\pi i}\frac{1}{w^{m+2}}\frac{1}{h'_I}\frac{\partial h_I}{\partial\xi_i}, \quad C^{(I)}_{i,m} = \oint\frac{dw}{2\pi i}\frac{1}{w^{m+2}}\frac{1}{h'_I}\frac{\partial h_I}{\partial\bar{\xi}_i}, \tag{A.3}$$

with h_I being the maps from the local coordinates w_I at the puncture I to the uniformizer z on the sphere

$$z = h_I(w_I;\xi_1,\bar{\xi}_1,\xi_2,\bar{\xi}_2) = z_I + \rho_I w_I + \rho_I^2\beta_I w_I^2 + \rho_I^3\gamma_I w_I^3 + \mathcal{O}(w_I^4). \tag{A.4}$$

All the coefficients in the right-hand side depend on the complex numbers $\xi_1 = x_1 + iy_1$ and $\xi_2 = x_2 + iy_2$ that parameterize the five-punctured spheres, and the z_I are the punctures, where the states are inserted, $z_1 = 0$, $z_2 = 1$, $z_3 = \xi_1$, $z_4 = \xi_2$. The fifth puncture is at $z = \infty$ and there we should use the coordinate $t = 1/z$

$$t = h_5(w_5;\xi_1,\bar{\xi}_1,\xi_2,\bar{\xi}_2) = \rho_5 w_5 + \rho_5^2\beta_5 w_5^2 + \rho_5^3\gamma_5 w_5^3 + \mathcal{O}(w_5^4). \tag{A.5}$$

All these coefficients can be expressed in terms of the quadratic differential defining the geometry of the punctured sphere, which can in turn be expressed numerically in terms of ξ_1 and ξ_2 (see [9]).

In [9], the five states $|\Psi_i\rangle$ in (A.1) where the same and the integral could simply be written as 120 times the integral of the same function over $\mathcal{A}_5$. We now want to deal with the case where the states $|\Psi_i\rangle$ are different. We start by defining

$$F(\Psi_1, \Psi_2, \Psi_3|\Psi_4, \Psi_5) \equiv \langle\Sigma|\left(\mathcal{B}\mathcal{B}^\star\right)_1\left(\mathcal{B}\mathcal{B}^\star\right)_2|\Psi_1\rangle|\Psi_2\rangle|\Psi_4\rangle|\Psi_5\rangle|\Psi_3\rangle. \tag{A.6}$$

Note how we have separated the states Ψ_4 and Ψ_5 from the other ones. These are inserted on the punctures $z = \xi_1$ and $z = \xi_2$ respectively. The construction of the reduced moduli space done in [9] was such that these punctures always are on triangular faces of the interaction polyhedron, whereas the other three punctures $z = 0$, $z = 1$ and $z = \infty$ are always on quadrilateral faces. This is convenient because it makes visible the symmetry under the six $\mathrm{PSL}(2,\mathbb{Z})$ maps that permute the points 0, 1 and ∞. It will be convenient to explicitly name these maps

$$s_1(z) = z, \quad s_2(z) = \frac{1}{z}, \quad s_3(z) = 1 - z, \quad s_4(z) = \frac{1}{1-z}, \quad s_5(z) = \frac{z-1}{z}, \quad s_6(z) = \frac{z}{z-1}. \tag{A.7}$$

We can then write

$$\int_{\mathcal{V}_{0,5}} dx_1 dy_1 dx_2 dy_2 \langle\Sigma|\left(\mathcal{B}\mathcal{B}^\star\right)_1\left(\mathcal{B}\mathcal{B}^\star\right)_2|\Psi_1\rangle|\Psi_2\rangle|\Psi_3\rangle|\Psi_4\rangle|\Psi_5\rangle =$$
$$= \sum_{i=1}^{6}\left(\int_{s_i(\mathcal{A}_5)} + \int_{\overline{s_i(\mathcal{A}_5)}}\right)\Big(F(\Psi_1, \Psi_2, \Psi_3|\Psi_4, \Psi_5) + \text{permutations}\Big) dx_1 dy_1 dx_2 dy_2, \tag{A.8}$$

where the permutations denote the ten different ways of assigning three states to the first three arguments of F regardless of order. In other words those are the ten different ways of assigning three states to the quadrilateral faces. The integrals over the complex conjugates $\overline{s_i(\mathcal{A}_5)}$ can be easily related to the integrals over $s_i(\mathcal{A}_5)$ after we note that the parameters a_i of the quadratic differentials (see [9]) obey $a_i(\overline{\xi_1}, \overline{\xi_2}) = \overline{a_i}(\xi_1, \xi_2)$, $i = 1, 2$. We simply have

$$\int_{\overline{s_i(\mathcal{A}_5)}} F(\Psi_1, \Psi_2, \Psi_3|\Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 = \int_{s_i(\mathcal{A}_5)} \overline{F(\Psi_1, \Psi_2, \Psi_3|\Psi_4, \Psi_5)} dx_1 dy_1 dx_2 dy_2.$$

And since our states always obey the reality condition, we have $F = \overline{F}$ on the Hilbert spaces we are considering in this paper. Thus

$$\int_{\overline{s_i(\mathcal{A}_5)}} F(\Psi_1, \Psi_2, \Psi_3|\Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 = \int_{s_i(\mathcal{A}_5)} F(\Psi_1, \Psi_2, \Psi_3|\Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2. \tag{A.9}$$

For (A.8) to make sense we still need to show that the order of the first three arguments and the order of the last two arguments do not matter in the expression

$$\sum_{i=1}^{6}\int_{s_i(\mathcal{A}_5)} F(\Psi_1, \Psi_2, \Psi_3|\Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2.$$

To show that, we first remind that, in [9], we defined the space $\mathcal{V}_{0,5}^{\{0,1,\infty\}}$ to be the subspace of $\mathcal{V}_{0,5}$ for which the punctures at 0, 1 and ∞ are on quadrilateral faces. It can be written

$$\mathcal{V}_{0,5}^{\{0,1,\infty\}} = \bigcup_{i=1}^{6}\left(s_i(\mathcal{A}_5) \cup \overline{s_i(\mathcal{A}_5)}\right). \tag{A.10}$$

From its definition, this space is obviously symmetric under the exchange $\xi_1 \leftrightarrow \xi_2$, which corresponds to the exchange of the last two arguments of F. Therefore we have, using (A.9) and (A.10)

$$\sum_{i=1}^{6} \int_{s_i(\mathcal{A}_5)} F(\Psi_1, \Psi_2, \Psi_3 | \Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 = \sum_{i=1}^{6} \int_{s_i(\mathcal{A}_5)} F(\Psi_1, \Psi_2, \Psi_3 | \Psi_5, \Psi_4) dx_1 dy_1 dx_2 dy_2. \tag{A.11}$$

At last, the integrations over $s_i(\mathcal{A}_5)$ can be written as integrals over $\mathcal{A}_5$ after permutations of the first three punctures. Namely

$$\begin{aligned}
\int_{s_2(\mathcal{A}_5)} F(\Psi_1, \Psi_2, \Psi_3 | \Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 &= \int_{\mathcal{A}_5} F(\Psi_3, \Psi_2, \Psi_1 | \Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 \\
\int_{s_3(\mathcal{A}_5)} F(\Psi_1, \Psi_2, \Psi_3 | \Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 &= \int_{\mathcal{A}_5} F(\Psi_2, \Psi_1, \Psi_3 | \Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 \\
\int_{s_4(\mathcal{A}_5)} F(\Psi_1, \Psi_2, \Psi_3 | \Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 &= \int_{\mathcal{A}_5} F(\Psi_2, \Psi_3, \Psi_1 | \Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 \\
\int_{s_5(\mathcal{A}_5)} F(\Psi_1, \Psi_2, \Psi_3 | \Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 &= \int_{\mathcal{A}_5} F(\Psi_3, \Psi_1, \Psi_2 | \Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 \\
\int_{s_6(\mathcal{A}_5)} F(\Psi_1, \Psi_2, \Psi_3 | \Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2 &= \int_{\mathcal{A}_5} F(\Psi_1, \Psi_3, \Psi_2 | \Psi_4, \Psi_5) dx_1 dy_1 dx_2 dy_2
\end{aligned} \tag{A.12}$$

Now from (A.1), (A.8), (A.9), (A.11) and (A.12) we can simply write

$$\boxed{\{\Psi_1, \Psi_2, \Psi_3, \Psi_4, \Psi_5\} = \frac{1}{\pi^2} \sum_{\sigma \in S_5} \int_{\mathcal{A}_5} F(\Psi_{\sigma(1)}, \Psi_{\sigma(2)}, \Psi_{\sigma(3)} | \Psi_{\sigma(4)}, \Psi_{\sigma(5)}) dx_1 dy_1 dx_2 dy_2}, \tag{A.13}$$

where we sum over all the 120 elements of the permutation group S_5 of five elements. This is the most symmetric way of writing the multilinear function as an integral over $\mathcal{A}_5$. We now want to specialize this formula for the two special cases encountered in this paper, when we have only two different kinds of states.

Now assume that we have $\Psi_1 = \ldots = \Psi_4 = \Phi$ and $\Psi_5 = \Psi$. From (A.13) we can write

$$\begin{aligned}
\kappa^2 V_{\phi^4 \psi} = \frac{1}{24} \{\Psi, \Phi, \Phi, \Phi, \Phi\} = & \frac{1}{\pi^2} \int_{\mathcal{A}_5} \Big(F(\Psi, \Phi, \Phi | \Phi, \Phi) + F(\Phi, \Psi, \Phi | \Phi, \Phi) + F(\Phi, \Phi, \Psi | \Phi, \Phi) \\
& + F(\Phi, \Phi, \Phi | \Psi, \Phi) + F(\Phi, \Phi, \Phi | \Phi, \Psi) \Big) dx_1 dy_1 dx_2 dy_2.
\end{aligned} \tag{A.14}$$

From (A.11) and (A.12), we have that

$$\int_{\mathcal{A}_5} F(\Phi, \Phi, \Phi | \Psi, \Phi) dx_1 dy_1 dx_2 dy_2 = \int_{\mathcal{A}_5} F(\Phi, \Phi, \Phi | \Phi, \Psi) dx_1 dy_1 dx_2 dy_2, \tag{A.15}$$

Introducing the definition

$$F^{(I)}_{\phi^4 \psi} \equiv \langle \Sigma | \left(\mathcal{B}\mathcal{B}^\star\right)_1 \left(\mathcal{B}\mathcal{B}^\star\right)_2 | \Psi^{(I)} \Phi^{(J)} \Phi^{(K)} \Phi^{(L)} \Phi^{(H)} \rangle \tag{A.16}$$

where the state Ψ is inserted on the puncture I, and the Φ's are inserted on the other four punctures J, K, L and H. We can now write

$$\boxed{\kappa^2 V_{\phi^4\psi} = \frac{1}{\pi^2}\int_{\mathcal{A}_5}\left(F^{(1)}_{\phi^4\psi} + F^{(2)}_{\phi^4\psi} + F^{(5)}_{\phi^4\psi} + 2F^{(3)}_{\phi^4\psi}\right) dx_1 dy_1 dx_2 dy_2}. \tag{A.17}$$

This is not the most symmetric way to write the amplitude, but it involves less different functions $F^{(I)}_{\phi^4\psi}$, which are quite long expressions that take time to calculate.

Next we assume that $\Psi_1 = \ldots = \Psi_3 = \Phi$ and $\Psi_4 = \Psi_5 = \Psi$. This time we have

$$\kappa^2 V_{\phi^3\psi^2} = \frac{1}{12}\left\{\Psi, \Psi, \Phi, \Phi, \Phi\right\}. \tag{A.18}$$

Again we can use (A.11) to reduce a little bit the number of different functions in the integral, noting that

$$\begin{aligned}
&\int_{\mathcal{A}_5}\left(F(\Psi,\Phi,\Phi|\Psi,\Phi) + F(\Phi,\Psi,\Phi|\Psi,\Phi) + F(\Phi,\Phi,\Psi|\Psi,\Phi)\right) dx_1 dy_1 dx_2 dy_2 = \\
&= \int_{\mathcal{A}_5}\left(F(\Psi,\Phi,\Phi|\Phi,\Psi) + F(\Phi,\Psi,\Phi|\Phi,\Psi) + F(\Phi,\Phi,\Psi|\Phi,\Psi)\right) dx_1 dy_1 dx_2 dy_2 .
\end{aligned}$$

Extending the definition (A.16) with

$$F^{(IJ)}_{\phi^3\psi^2} \equiv \langle\Sigma|\left(\mathcal{B}\mathcal{B}^\star\right)_1\left(\mathcal{B}\mathcal{B}^\star\right)_2|\Psi^{(I)}\Psi^{(J)}\Phi^{(K)}\Phi^{(L)}\Phi^{(H)}\rangle, \tag{A.19}$$

where the two states Ψ are inserted at the punctures I and J and the states Φ are inserted at the other three punctures K, L and H, we find

$$\boxed{\kappa^2 V_{\phi^3\psi^2} = \frac{1}{\pi^2}\int_{\mathcal{A}_5}\left(F^{(12)}_{\phi^3\psi^2} + F^{(15)}_{\phi^3\psi^2} + F^{(25)}_{\phi^3\psi^2} + F^{(34)}_{\phi^3\psi^2} + 2F^{(13)}_{\phi^3\psi^2} + 2F^{(23)}_{\phi^3\psi^2} + 2F^{(53)}_{\phi^3\psi^2}\right) dx_1 dy_1 dx_2 dy_2}. \tag{A.20}$$

A.2 Contact terms of tachyons and dilatons

We now list the results for the functions F that we used in this paper. The results of the integrations are shown in Tables 1, 6 and 7. We start with the terms with tachyons and dilatons. The five-tachyon and five-dilaton terms were calculated in [9] so we don't repeat them here. We will need the following open ghost correlators

$$\begin{aligned}
&A_{IJ} \equiv \langle (c_{-1}c_1)^{(I)}, c^{(J)}_{-1}\rangle_o\,, \quad B_{IJ} \equiv \langle (c_{-1}c_1)^{(I)}, c^{(J)}_{1}\rangle_o \\
&C_{IJK} \equiv \langle c^{(I)}_1, c^{(J)}_1, c^{(K)}_1\rangle_o\,, \quad D_{IJK} \equiv \langle c^{(I)}_{-1}, c^{(J)}_1, c^{(K)}_1\rangle_o\,, \quad E_{IJK} \equiv \langle c^{(I)}_{-1}, c^{(J)}_{-1}, c^{(K)}_1\rangle_o .
\end{aligned} \tag{A.21}$$

Expressed in terms of the coefficients in the maps expansions (A.4) and (A.5), these are (defining $z_{IJ} \equiv z_I - z_J$ and $\epsilon_I \equiv 8\beta_I^2 - 6\gamma_I$)

$$
\begin{aligned}
A_{IJ} &= \rho_J \left(\beta_J - \beta_I - 2\beta_I \beta_J z_{IJ} + \frac{1}{2}\epsilon_J z_{IJ}(1-\beta_I z_{IJ}) \right), \quad A_{5J} = \rho_J \left(\frac{1}{2}\epsilon_J(\beta_5 + z_J) - \beta_J \right) \\
B_{IJ} &= \frac{1}{\rho_J} z_{IJ}(1-\beta_I z_{IJ}), \quad B_{I5} = \frac{\beta_I}{\rho_5}, \quad B_{5J} = \frac{1}{\rho_J}(z_J + \beta_5) \\
C_{IJK} &= \frac{1}{\rho_I \rho_J \rho_K} z_{IJ} z_{IK} z_{JK}, \quad C_{IJ5} = \frac{z_{JI}}{\rho_I \rho_J \rho_5} \\
D_{IJK} &= \frac{\rho_I}{\rho_J \rho_K} \left(z_{JK} - \beta_I (z_{IK} + z_{IJ}) z_{JK} + \frac{1}{2}\epsilon_I z_{IJ} z_{JK} z_{IK} \right), \quad D_{IJ5} = \frac{\rho_I}{\rho_J \rho_5} \left(\beta_I - \frac{1}{2}\epsilon_I z_{IJ} \right), \\
&\quad D_{5IJ} = \frac{\rho_5}{\rho_I \rho_J} z_{JI} \left(z_I z_J + \beta_5 (z_I + z_J) + \frac{\epsilon_5}{2} \right) \\
E_{IJK} &= \frac{\rho_I \rho_J}{\rho_K} \left(\beta_I - \beta_J + \beta_I \beta_J (z_{IJ} + z_{IK} - z_{JK}) + \frac{1}{2}\epsilon_J z_{JK} - \frac{1}{2}\beta_I \epsilon_J (z_{IJ} + z_{IK}) z_{JK} + \right. \\
&\quad \left. + \frac{1}{2}\epsilon_I \left(-z_{IK} + \beta_J z_{IK}(z_{JK} - z_{IJ}) + \frac{1}{2}\epsilon_J z_{IJ} z_{IK} z_{JK} \right) \right), \\
&\quad E_{IJ5} = \frac{\rho_I \rho_J}{\rho_5} \left(\frac{1}{2}\beta_I \epsilon_J - \frac{1}{2}\epsilon_I \beta_J - \frac{1}{4}\epsilon_I \epsilon_J z_{IJ} \right),
\end{aligned} \tag{A.22}
$$

and it is understood that $I, J, K \neq 5$. We can now present the results for the closed correlators.

Four tachyons and one dilaton

$$
F^{(3)}_{t^4 d} = 4\,\mathrm{Re} \left(\frac{C^3_{1,1}}{\rho_1^2 \rho_2^2 \rho_3 \rho_4^2 \rho_5^2} \right) \tag{A.23}
$$

$$
F^{(I)}_{t^4 d} = 4\,\mathrm{Re} \left(\frac{C_{IJK}}{\rho_3 \rho_4} \left(\frac{C^I_{2,1} \overline{C_{J4K}}}{\rho_3} + \frac{C^I_{1,1} \overline{C_{J3K}}}{\rho_4} \right) \right), \quad I \neq J \neq K \neq 3 \neq 4. \tag{A.24}
$$

Note that we are giving a transitive meaning to the inequality sign. So for example, by $I \neq J \neq K \neq 3 \neq 4$ we really mean that I, J and K are pairwise distinct and that none of them is equal to 3 or 4. For a given I, there are two possible choices of J and K in equation (A.24), but they give the same result because the right-hand side of (A.24) is manifestly invariant under $J \leftrightarrow K$.

Three tachyons and two dilatons

$$
F^{(34)}_{t^3 d^2} = 4 \frac{|C_{125}|^2}{\rho_3 \rho_4} \mathrm{Re} \left\{ C^3_{1,1} C^4_{2,1} - C^4_{1,1} C^3_{2,1} + C^3_{1,1} \overline{C^4_{2,1}} - \overline{B^4_{1,1}} B^3_{2,1} \right\} \tag{A.25}
$$

$$
\begin{aligned}
F^{(I3)}_{t^3 d^2} = 2\,\mathrm{Re} \sum_{\substack{J,K \\ J \neq K \neq I \neq 3 \neq 4}} & \left\{ C_{IJK} \left(\frac{\overline{C_{J4K}}}{\rho_3 \rho_4} \left(C^3_{1,1} C^I_{2,1} - C^I_{1,1} C^3_{2,1} + \overline{C^3_{1,1}} C^I_{2,1} - B^I_{1,1} \overline{B^3_{2,1}} \right) \right. \right. \\
& \left. \left. + \frac{\overline{C_{J3K}}}{\rho_4^2} \left(C^I_{1,1} \overline{C^3_{1,1}} - \overline{B^3_{1,1}} B^I_{1,1} \right) - \frac{\overline{D_{3JK}}}{\rho_3 \rho_4^2} B^I_{1,1} \right) \right\}, \quad I \neq 3, 4
\end{aligned} \tag{A.26}
$$

$$
F^{(JK)}_{t^3 d^2} = 4\,\mathrm{Re} \left\{ \frac{1}{\rho_3 \rho_4} \left(C^J_{1,1} C^K_{2,1} - C^K_{1,1} C^J_{2,1} \right) C_{IJK} \overline{C_{I34}} + \frac{1}{\rho_3^2 \rho_4^2} \overline{B_{KI}} B_{JI} \right.
$$

$$
\begin{aligned}
&- \frac{1}{\rho_3^2\rho_4}\left(\overline{B^K_{2,1}}B_{JI}\overline{C_{I4K}} - B^J_{2,1}C_{IJ4}\overline{B_{KI}}\right) + \frac{1}{\rho_3\rho_4^2}\left(B^J_{1,1}C_{IJ3}\overline{B_{KI}} - \overline{B^K_{1,1}}B_{JI}\overline{C_{I3K}}\right)\\
&- \frac{1}{\rho_3^2}\left(B^J_{2,1}\overline{B^K_{2,1}} - \overline{C^K_{2,1}}C^J_{2,1}\right)C_{IJ4}\overline{C_{I4K}} - \frac{1}{\rho_4^2}\left(B^J_{1,1}\overline{B^K_{1,1}} - \overline{C^K_{1,1}}C^J_{1,1}\right)C_{IJ3}\overline{C_{I3K}}\\
&- \frac{1}{\rho_3\rho_4}\left(\overline{B^K_{1,1}}B^J_{2,1} - C^J_{1,1}\overline{C^K_{2,1}}\right)C_{IJ4}\overline{C_{I3K}}\\
&- \frac{1}{\rho_3\rho_4}\left(B^J_{1,1}\overline{B^K_{2,1}} - \overline{C^K_{1,1}}C^J_{2,1}\right)C_{IJ3}\overline{C_{I4K}}\Bigg\}, \quad I \neq J \neq K \neq 3 \neq 4.
\end{aligned}
\tag{A.27}
$$

Two tachyons and three dilatons

$$
\begin{aligned}
F^{(34)}_{t^2d^3} =& 4\,\mathrm{Re}\sum_{I\neq J\neq K\neq 3\neq 4}\Bigg\{\frac{1}{\rho_3\rho_4}C^I_{1,1}\left(\overline{B^J_{2,1}}B_{KI}\overline{C_{J34}} - B^J_{2,1}\overline{B_{K3}}C_{IJ4}\right)\\
&+ \frac{1}{\rho_3\rho_4}C^I_{2,1}\left(\overline{B^J_{1,1}}B_{KI}\overline{C_{J43}} - B^J_{1,1}\overline{B_{K4}}C_{IJ3}\right) + \frac{1}{\rho_3}C_{IJ4}\overline{C_{34K}}\left(C^I_{1,1}M^{JK}_2 + C^J_{2,1}B^I_{2,1}\overline{B^K_{1,1}}\right)\\
&+ \frac{1}{\rho_4}C_{IJ3}\overline{C_{43K}}\left(C^I_{2,1}M^{JK}_1 + C^J_{1,1}B^I_{1,1}\overline{B^K_{2,1}}\right) - \frac{1}{\rho_3\rho_4}\left(\frac{1}{\rho_4}C^I_{1,1}B_{JI}\overline{B_{K3}} + \frac{1}{\rho_3}C^I_{2,1}B_{JI}\overline{B_{K4}}\right)\\
&+ \frac{1}{\rho_4^2}B^I_{1,1}C^J_{1,1}C_{IJ3}\overline{B_{K3}} + \frac{1}{\rho_3^2}B^I_{2,1}C^J_{2,1}C_{IJ4}\overline{B_{K4}}\Bigg\}
\end{aligned}
\tag{A.28}
$$

$$
\begin{aligned}
F^{(JK)}_{t^2d^3} =& 4\,\mathrm{Re}\Bigg\{\overline{C_{IJK}}\Bigg\{\frac{1}{\rho_3\rho_4}\left(C^4_{1,1}\overline{B^I_{2,1}} - C^4_{2,1}\overline{B^I_{1,1}}\right)D_{3JK} - \frac{1}{\rho_3\rho_4}\left(C^3_{1,1}\overline{B^I_{2,1}} - C^3_{2,1}\overline{B^I_{1,1}}\right)D_{4JK}\\
&+ \frac{1}{\rho_3}\Bigg(C^4_{1,1}\left(B^3_{2,1}\overline{B^I_{2,1}} - C^3_{2,1}\overline{C^I_{2,1}}\right) - C^3_{1,1}\left(B^4_{2,1}\overline{B^I_{2,1}} - C^4_{2,1}\overline{C^I_{2,1}}\right)\\
&\qquad -\overline{B^I_{1,1}}\left(B^3_{2,1}C^4_{2,1} - B^4_{2,1}C^3_{2,1}\right)\Bigg)C_{J4K}\\
&+ \frac{1}{\rho_4}\Bigg(C^3_{2,1}\left(B^4_{1,1}\overline{B^I_{1,1}} - C^4_{1,1}\overline{C^I_{1,1}}\right) - C^4_{2,1}\left(B^3_{1,1}\overline{B^I_{1,1}} - C^3_{1,1}\overline{C^I_{1,1}}\right)\\
&\qquad -\overline{B^I_{2,1}}\left(B^4_{1,1}C^3_{1,1} - B^3_{1,1}C^4_{1,1}\right)\Bigg)C_{J3K}\\
&+ \frac{1}{\rho_3}\Bigg(\overline{C^I_{1,1}}\left(B^4_{2,1}\overline{B^3_{2,1}} - C^4_{2,1}\overline{C^3_{2,1}}\right) - \overline{C^3_{1,1}}\left(B^4_{2,1}\overline{B^I_{2,1}} - C^4_{2,1}\overline{C^I_{2,1}}\right)\\
&\qquad -B^4_{1,1}\left(\overline{B^3_{2,1}C^I_{2,1}} - \overline{B^I_{2,1}C^3_{2,1}}\right)\Bigg)C_{J4K}\\
&+ \frac{1}{\rho_4}\Bigg(\overline{C^I_{2,1}}\left(B^3_{1,1}\overline{B^4_{1,1}} - C^3_{1,1}\overline{C^4_{1,1}}\right) - \overline{C^4_{2,1}}\left(B^3_{1,1}\overline{B^I_{1,1}} - C^3_{1,1}\overline{C^I_{1,1}}\right)\\
&\qquad -B^3_{2,1}\left(\overline{B^4_{1,1}C^I_{1,1}} - \overline{B^I_{1,1}C^4_{1,1}}\right)\Bigg)C_{J3K} + \frac{1}{\rho_3\rho_4}\left(\overline{C^I_{1,1}B^3_{2,1}} - \overline{C^3_{1,1}B^I_{2,1}}\right)D_{4JK}\\
&+ \frac{1}{\rho_3\rho_4}\left(\overline{C^I_{2,1}B^4_{1,1}} - \overline{C^4_{2,1}B^I_{1,1}}\right)D_{3JK}\Bigg\}\Bigg\}, \quad I \neq J \neq K \neq 3 \neq 4
\end{aligned}
\tag{A.29}
$$

$$
\begin{aligned}
F^{(I3)}_{t^2d^3} =& 4\,\mathrm{Re}\Bigg\{\sum_{\substack{J,K,L\\ J\neq K\neq L\neq 3\neq I}}\left\{\frac{1}{\rho_3}C_{JKI}\overline{C_{3IL}}\left(C^J_{1,1}M^{KL}_2 + C^K_{2,1}B^J_{2,1}\overline{B^L_{1,1}}\right) + \frac{1}{\rho_3^2}B^J_{2,1}C^K_{2,1}C_{JKI}\overline{B_{LI}}\right\}\\
&+ \frac{1}{\rho_4}\sum_{\substack{J,K\\ J\neq K\neq I\neq 3\neq 4}}\Bigg\{\Bigg(C^J_{2,1}\left(B^4_{1,1}\overline{B^K_{1,1}} - C^4_{1,1}\overline{C^K_{1,1}}\right) - C^4_{2,1}\left(B^J_{1,1}\overline{B^K_{1,1}} - C^J_{1,1}\overline{C^K_{1,1}}\right)\\
&\qquad -\overline{B^K_{2,1}}\left(B^4_{1,1}C^J_{1,1} - B^J_{1,1}C^4_{1,1}\right)\Bigg)C_{IJ3}\overline{C_{KI3}} + \frac{1}{\rho_3^2}\left(C^4_{2,1}B_{JI} + C^J_{2,1}D_{4IJ}\right)\overline{B_{KI}}
\end{aligned}
$$

$$
+\frac{1}{\rho_3}\left(B^4_{1,1}C^K_{2,1}-B^K_{1,1}C^4_{2,1}\right)C_{I3K}\overline{B_{JI}}+\frac{1}{\rho_3}\left(B^J_{1,1}\overline{C^4_{2,1}}-B^J_{2,1}\overline{C^4_{1,1}}\right)C_{IJ3}\overline{B_{KI}}
$$

$$
+\frac{1}{\rho_3}\left(B^J_{1,1}\overline{C^K_{2,1}}-B^J_{2,1}\overline{C^K_{1,1}}\right)C_{IJ3}\overline{D_{4IK}}+\frac{1}{\rho_3}C^J_{1,1}B^K_{2,1}C_{IJK}\overline{D_{4I3}}\Bigg\}\Bigg\},\quad I\neq 3,4. \tag{A.30}
$$

One tachyon and four dilatons

$$
\begin{aligned}
F^{(3)}_{td^4}=&4\,\mathrm{Re}\Bigg\{\sum_{I\neq J\neq K\neq L\neq 3}\Bigg\{\frac{1}{2\rho_3^2}M_2^{IJ}B_{KI}\overline{B_{LJ}}+\frac{1}{\rho_3}\left(C^I_{1,1}\overline{B^J_{2,1}C^K_{2,1}}+\overline{B^J_{1,1}}M_2^{IK}\right)B_{LI}\overline{C_{J3K}}\\
&-\frac{1}{\rho_3}C^I_{1,1}B^J_{2,1}C^K_{2,1}\overline{B_{L3}}C_{IJK}+B^I_{1,1}C^J_{1,1}\overline{B^K_{2,1}C^L_{2,1}}C_{IJ3}\overline{C_{3KL}}+\frac{1}{2}M_1^{IJ}M_2^{KL}C_{I3K}\overline{C_{J3L}}\Bigg\}\\
&+\sum_{I\neq J\neq K\neq 3\neq 4}\Bigg\{-\frac{1}{\rho_3^2\rho_4}\overline{B^I_{2,1}}A_{J4}\overline{B_{KI}}+\frac{1}{\rho_3\rho_4}\left(C^I_{1,1}\overline{C^J_{2,1}}-\overline{B^J_{1,1}}B^I_{2,1}\right)B_{KI}\overline{D_{4J3}}\\
&+\frac{1}{\rho_3\rho_4}\left(\left(\overline{B^4_{1,1}}B^J_{2,1}+C^4_{1,1}C^J_{2,1}-C^J_{1,1}C^4_{2,1}-C^J_{1,1}\overline{C^4_{2,1}}\right)B_{IJ}\overline{B_{K3}}-C^I_{1,1}C^J_{2,1}\overline{B_{K3}}D_{4IJ}\right)\\
&+\frac{1}{\rho_3\rho_4}\overline{B^I_{1,1}B^J_{2,1}}A_{K4}\overline{C_{IJ3}}+\frac{1}{\rho_4}B^I_{1,1}C^J_{1,1}C_{IJ3}\left(\overline{C^K_{2,1}D_{43K}}+\overline{C^4_{2,1}B_{K3}}\right)\\
&+\frac{1}{\rho_4}\left(B^I_{1,1}C^J_{1,1}C^4_{2,1}-B^I_{1,1}C^4_{1,1}C^J_{2,1}+B^4_{1,1}C^I_{1,1}C^J_{2,1}\right)C_{IJ3}\overline{B_{K3}}\\
&+\frac{1}{\rho_4}\left(-M_1^{IJ}\overline{B^K_{2,1}}D_{4I3}\overline{C_{J3K}}+M_1^{4J}\overline{B^K_{2,1}}B_{I3}\overline{C_{J3K}}\right)\Bigg\}\Bigg\}
\end{aligned}\tag{A.31}
$$

$$
\begin{aligned}
F^{(I)}_{td^4}=&4\,\mathrm{Re}\Bigg\{\frac{1}{\rho_4}\sum_{\substack{J,K,L\\ J\neq K\neq L\neq I\neq 4}}\Bigg\{B^J_{1,1}C^K_{11}C_{JKI}\left(C^L_{2,1}\overline{D_{4IL}}+\overline{C^4_{2,1}B_{LI}}\right)-M_1^{JK}\overline{B^L_{2,1}}D_{4JI}\overline{C_{KIL}}\\
&+M_1^{4K}\overline{B^L_{2,1}}B_{JI}\overline{C_{KIL}}+\left(B^J_{1,1}C^K_{1,1}C^4_{2,1}-B^J_{1,1}C^4_{1,1}C^K_{2,1}+B^4_{1,1}C^J_{1,1}C^K_{2,1}\right)C_{JKI}\overline{B_{LI}}\Bigg\}\\
&+\sum_{\substack{J,K,L,H\\ J\neq K\neq L\neq H\neq I}}\Bigg\{B^J_{1,1}C^K_{1,1}\overline{B^L_{2,1}C^H_{2,1}}C_{JKI}\overline{C_{ILH}}+\frac{1}{2}M_1^{JK}M_2^{LH}C_{JIL}\overline{C_{KIH}}\Bigg\}\\
&+\frac{1}{\rho_3\rho_4}\sum_{\substack{J,K\\ J\neq K\neq I\neq 3\neq 4}}\Bigg\{\left(C^J_{1,1}\overline{C^K_{2,1}}-B^J_{2,1}\overline{B^K_{1,1}}\right)D_{3IJ}\overline{D_{4IK}}+\overline{B^J_{1,1}}B^K_{2,1}E_{34I}\overline{C_{IJK}}\\
&+\left(\overline{C^3_{1,1}}C^K_{2,1}-\overline{B^3_{2,1}}B^K_{1,1}+C^3_{1,1}C^K_{2,1}-C^3_{2,1}C^K_{1,1}\right)D_{4IK}\overline{B_{JI}}\\
&+\left(\overline{C^4_{2,1}}C^K_{1,1}-\overline{B^4_{1,1}}B^K_{2,1}+C^4_{2,1}C^K_{1,1}-C^4_{1,1}C^K_{2,1}\right)D_{3IK}\overline{B_{JI}}\\
&+\left(\overline{C^4_{2,1}}C^3_{1,1}-\overline{B^4_{1,1}}B^3_{2,1}+C^4_{2,1}C^3_{1,1}-C^4_{1,1}C^3_{2,1}\right)B_{JI}\overline{B_{KI}}\Bigg\}\\
&+\frac{1}{\rho_3}\sum_{\substack{J,K,L\\ J\neq K\neq L\neq I\neq 3}}C_{IKL}\Bigg\{\left(\overline{C^J_{1,1}}B^K_{2,1}C^L_{2,1}+B^K_{1,1}\overline{M_2^{JL}}\right)\overline{D_{3IJ}}+\left(\overline{C^3_{1,1}}B^K_{2,1}C^L_{2,1}+B^K_{1,1}\overline{M_2^{3L}}\right.\\
&\left.+C^3_{1,1}B^K_{2,1}C^L_{2,1}-C^K_{1,1}B^3_{2,1}C^L_{2,1}-C^L_{1,1}B^K_{2,1}C^3_{2,1}\right)\overline{B_{JI}}\Bigg\}\Bigg\},\quad I\neq 3,4
\end{aligned}\tag{A.32}
$$

The results of the integrations are shown in Table 6.

A.3 Contact terms of dilatons and marginal fields

Since the ghost part of the marginal state $|A\rangle$, defined in (2.2), is that of a tachyon, and because the correlators factorize into ghost and matter parts, we can recycle the results of correlators of tachyons and dilatons. We just need to calculate the matter correlators. For the correlators of two marginals and three dilatons, we have

$$F^{(IJ)}_{a^2d^3} = F^{(IJ)}_{t^2d^3} \left| \langle\langle \alpha^{(I)}_{-1}\alpha^{(J)}_{-1} \rangle\rangle_o \right|^2, \quad I, J = 1, \ldots, 5, \tag{A.33}$$

where the open matter correlators $\langle\langle \ldots \rangle\rangle_o$ are

$$\langle\langle \alpha^{(I)}_{-1}\alpha^{(J)}_{-1} \rangle\rangle_o = \frac{\rho_I \rho_J}{z_{JI}}, \quad \langle\langle \alpha^{(I)}_{-1}\alpha^{(5)}_{-1} \rangle\rangle_o = -\rho_I \rho_5, \quad I, J = 1, \ldots, 4. \tag{A.34}$$

And for the correlators of four marginal fields and one dilaton we have

$$F^{(I)}_{a^4d} = F^{(I)}_{t^4d} \left| \langle\langle \alpha^{(J)}_{-1}\alpha^{(K)}_{-1}\alpha^{(L)}_{-1}\alpha^{(H)}_{-1} \rangle\rangle_o \right|^2, \quad I = 1, \ldots, 5, \quad J \neq K \neq L \neq H \neq I. \tag{A.35}$$

For these matter correlators we find

$$\begin{aligned}
\langle\langle \alpha^{(I)}_{-1}\alpha^{(J)}_{-1}\alpha^{(K)}_{-1}\alpha^{(5)}_{-1} \rangle\rangle_o &= -\rho_I\rho_J\rho_K\rho_5 \left(\frac{1}{z_{IJ}^2} + \frac{1}{z_{IK}^2} + \frac{1}{z_{JK}^2} \right), \quad I, J, K \neq 5 \\
\langle\langle \alpha^{(1)}_{-1}\alpha^{(2)}_{-1}\alpha^{(3)}_{-1}\alpha^{(4)}_{-1} \rangle\rangle_o &= \rho_1\rho_2\rho_3\rho_4 \left(\frac{1}{(\xi_1 - \xi_2)^2} + \frac{1}{\xi_1^2(1-\xi_2)^2} + \frac{1}{\xi_2^2(1-\xi_1)^2} \right).
\end{aligned} \tag{A.36}$$

The results of the integrations are shown in Table 1.

A.4 Contact terms of four tachyons and one field of level four

The level four fields f_1, f_2, f_3 and g_1 were defined in (5.1). We will need a few new open correlators. We define

$$P_{IJKL} \equiv \langle b_{-2}c^{(I)}_1, c^{(J)}_1, c^{(K)}_1, c^{(L)}_1 \rangle_o, \quad Q_3 \equiv \langle c^{(1)}_1, c^{(2)}_1, b^{(3)}_{-2}, c^{(4)}_1, c^{(5)}_1 \rangle_o, \quad G_I \equiv \langle\langle L^{(I)}_{-2} \rangle\rangle_o. \tag{A.37}$$

Elementary calculations give the following expressions for the correlators that we need

$$\begin{aligned}
P_{IJK5} =& \frac{\rho_I}{\rho_J\rho_K\rho_5} z_{JK} \left(\frac{1}{z_{IJ}} + \frac{1}{z_{IK}} - 3\beta_I \right) \\
P_{51IJ} =& \frac{\rho_5}{\rho_1\rho_I\rho_J} z_{IJ} \left(z_I z_J \left(1 + \xi_1 + \xi_2 + 3\beta_5\right) - \xi_1\xi_2 \right) \\
Q_3 =& \frac{\rho_3^2}{\rho_1\rho_2\rho_4\rho_5} \frac{\xi_2(1-\xi_2)}{\xi_1(1-\xi_1)(\xi_1-\xi_2)} \\
G_I =& \frac{13}{6}\rho_I^2 \left(2\beta_I^2 - \epsilon_I \right),
\end{aligned} \tag{A.38}$$

where $I, J, K = 1, \ldots, 4$. And for the functions to integrate we find

$$F^{(3)}_{t^4 f_1} = \frac{2}{\rho_4^2} \left(\left| B^3_{1,1} \right|^2 - \left| C^3_{1,1} \right|^2 \right) \left| C_{125} \right|^2 \tag{A.39}$$

$$\begin{aligned} F^{(I)}_{t^4 f_1} =& 4 \operatorname{Re} \left\{ \frac{\left| D_{IJK} \right|^2}{2\rho_3^2 \rho_4^2} + \frac{B^I_{1,1}}{\rho_3 \rho_4^2} C_{J3K} \overline{D_{IJK}} + \frac{B^I_{2,1}}{\rho_3^2 \rho_4} C_{J4K} \overline{D_{IJK}} \right. \\ & + \frac{\left| C_{J4K} \right|^2}{2\rho_3^2} \left(\left| B^I_{2,1} \right|^2 - \left| C^I_{2,1} \right|^2 \right) + \frac{\left| C_{J3K} \right|^2}{2\rho_4^2} \left(\left| B^I_{1,1} \right|^2 - \left| C^I_{1,1} \right|^2 \right) \\ & \left. + \frac{1}{\rho_3 \rho_4} \left(B^I_{1,1} \overline{B^I_{2,1}} - \overline{C^I_{1,1}} C^I_{2,1} \right) C_{J3K} \overline{C_{J4K}} \right\}, \quad I \neq J \neq K \neq 3 \neq 4 \end{aligned} \tag{A.40}$$

$$F^{(I)}_{t^4 f_2} = \frac{2}{\rho_3^2 \rho_4^2} \left| G_I \right|^2 \left| C_{125} \right|^2, \quad I = 1, \ldots, 5 \tag{A.41}$$

$$F^{(3)}_{t^4 f_3} = \frac{4}{\rho_3 \rho_4^2} \left| C_{125} \right|^2 \operatorname{Re} \left\{ G_3 \overline{B^3_{1,1}} \right\} \tag{A.42}$$

$$F^{(I)}_{t^4 f_3} = 4 \operatorname{Re} \left\{ \frac{G_I C_{IJK}}{\rho_3^2 \rho_4^2} \left(\overline{D_{IJK}} + \rho_3 \overline{B^I_{1,1} C_{J3K}} + \rho_4 \overline{B^I_{2,1} C_{J4K}} \right) \right\}, \quad I \neq J \neq K \neq 3 \neq 4 \tag{A.43}$$

$$F^{(3)}_{t^4 g_1} = 4 \operatorname{Re} \left\{ C_{125} \left(\frac{C^3_{1,2}}{\rho_3 \rho_4^2} \overline{P_{3125}} + \frac{C^3_{2,2}}{\rho_3^2 \rho_4} \overline{Q_3} \right) \right\} \tag{A.44}$$

$$F^{(I)}_{t^4 g_1} = 4 \operatorname{Re} \left\{ C_{IJK} \left(\frac{C^I_{1,2}}{\rho_3 \rho_4^2} \overline{P_{IJ3K}} + \frac{C^I_{2,2}}{\rho_3^2 \rho_4} \overline{P_{IJ4K}} \right) \right\}, \quad I \neq J \neq K \neq 3 \neq 4 \tag{A.45}$$

The results of the integrations are shown in Table 7.

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